### 4.2 Skill Builder

#### The Coordinate Grid

Plot $A(-2, 3)$ on a coordinate grid.
The first number in the ordered pair tells how far you move left or right on the horizontal axis.
The second number tells how far you move up or down on the vertical axis.
So, to plot $A(-2, 3)$:
Move 2 squares left of the origin, then move 3 squares up.

#### Check

1. What are the coordinates of points A, B, and C?

   | A(______) | B(______) | C(______) |

2. Graph these points on the coordinate grid.
   A(−3, 0) B(2, 4) C(0, −3)
Graphing Relations

This table of values shows how $2n + 1$ relates to $n$.

<table>
<thead>
<tr>
<th>Input, $n$</th>
<th>Output, $2n + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

The change in input is constant. The change in output is constant.

Use the data in the table to graph the relation.

The points lie on a straight line. This is a linear relation. The graph also shows how $2n + 1$ relates to $n$. On the graph, we see that each time the input increases by 1, the output increases by 2.

Check

1. a) Graph the data in this table of values.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

b) Is this a linear relation? Explain.
4.2 Linear Relations

**FOCUS** Analyze the graph of a linear relation.

A photographer charges $40 for a sitting fee, plus $20 per sheet of prints. The charges are shown in the table of values and in the graph.

<table>
<thead>
<tr>
<th>Number of Sheets, n</th>
<th>Cost, C ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
</tr>
</tbody>
</table>

We cannot order part of a sheet of prints. So, the points in the graph are not joined with a line. We say that the data are **discrete**.

For different values of $n$, we get different values of $C$. So the variable $C$ depends on the value of the variable $n$. When two variables are related in this way, they form a **relation**.

**Linear Relation**
When the graph of a relation is a straight line, it is called a **linear relation**.

---

**Example 1** Graphing a Linear Relation from a Table of Values

A popular DVD club allows members to purchase DVDs at a reduced price according to the table of values.

- **a)** Graph the data.
- **b)** Should the points be joined? Why or why not?
- **c)** Is the relation linear? Explain.
- **d)** Describe the patterns in the table. How are these patterns shown on the graph?

<table>
<thead>
<tr>
<th>Number of DVDs Purchased, n</th>
<th>Cost, C ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
</tr>
</tbody>
</table>
Solution

a) Plot the points on a grid.

b) The points should not be joined because you cannot buy part of a DVD.

c) The points on the graph lie on a straight line, so this is a linear relation.

d) The table of values shows that:
The number of DVDs purchased increases by 1 each time.
The cost increases by $5 each time.

To get from one point to the next in the graph, move 1 unit right and 5 units up.

Check

1. a) Graph the data from the table of values.

<table>
<thead>
<tr>
<th>Number of Floors, f</th>
<th>Building Height, h (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

b) Is the relation linear? Explain.

_________________________

_________________________

c) Should the points on the graph be joined with a line? Explain.

_________________________
Example 2  
Graphing a Linear Relation from an Equation

a) Complete the table of values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>y = 5 - x</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

b) Graph the relation represented by the data in the table of values.

![Graph of y = 5 - x](image)

Since we can substitute any value for x, the points can be joined with a line.

c) Describe the patterns in the graph and in the table.
Each point on the graph is 1 unit right and 1 unit down from the previous point. In the table, when x increases by 1, y decreases by 1.

d) Is the relation linear? Explain.
This is a linear relation because its graph is a straight line.

Solution

Check

1. Complete the table of values.
Then, graph the relation.

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th>y = 4x - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Which graphs represent a linear relation?

a) 

b) 

c) 

d) 

2. Describe the patterns in each table of values. Does each table of values represent a linear relation?

a) \[
\begin{array}{c|c}
 x & y \\
-3 & 6 \\
-2 & 5 \\
-1 & 4 \\
0 & 3 \\
\end{array}
\]

\[x \text{ increases by } \quad \text{each time.} \]
\[y \text{ decreases by } \quad \text{each time.} \]

The relation is _______, because a constant change in \( x \) produces a constant change in \( y \).

b) \[
\begin{array}{c|c}
 x & y \\
0 & 1 \\
2 & 4 \\
4 & 7 \\
6 & 10 \\
\end{array}
\]

\[x \text{ increases by } \quad \text{each time.} \]
\[y \text{ increases by } \quad \text{each time.} \]

The relation is _______, because a constant change in \( x \) produces a constant change in \( y \).

c) \[
\begin{array}{c|c}
 x & y \\
1 & 1 \\
2 & 3 \\
3 & 7 \\
4 & 13 \\
\end{array}
\]

\[x \text{ increases by } \quad \text{each time.} \]
\[y \]

The relation ________________
3. Each graph and table of values represents a linear relation.

a) Complete each table of values.

i)  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

ii)  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

b) Describe the patterns in the table.

i) When $x$ increases by ______, $y$ increases by ______.

ii) When ______ increases by ______, ______ increases by ______.

c) Describe the patterns in the graph.

i) To get from one point to the next, move 1 unit right and ______ up.

ii) To get from one point to the next, move ______ right and ______ up.

4. Complete the table of values for each linear relation, then graph it.

a) $y = 4x$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

b) $y = -3x$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

c) $y = 1 - x$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Complete the table of values. Graph the data.

\[
\begin{array}{c|c}
\hline
x & y = 2x - 4 \\
-1 & -6 \\
0 & -4 \\
1 & \hline
2 & \hline
\end{array}
\]

6. For special events, a bowling alley charges a set fee plus a fee for each hour bowled.

a) Graph the data.

**Bowling Costs**

<table>
<thead>
<tr>
<th>Hours, ( h )</th>
<th>Cost, ( C ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
</tr>
</tbody>
</table>

Does it make sense to join the points on the graph? Explain.

b) Is this a linear relation? Why?

c) Describe the pattern in words and using an equation.

When \( h \) increases by _____, \( C \) increases by _____.

\( C = ____ h + 30 \)
Lesson 4.2: Linear Relations

1. For each table of values below:
   i) Does it represent a linear relation?
   ii) If the relation is not linear, explain how you know.
   iii) If the relation is linear, describe it.

   a) \[ \begin{array}{cc}
   x & y \\
   1 & 5 \\
   2 & 12 \\
   3 & 19 \\
   4 & 26 \\
   5 & 33 \\
   \end{array} \]
   b) \[ \begin{array}{cc}
   x & y \\
   1 & 1 \\
   2 & 3 \\
   3 & 5 \\
   4 & 7 \\
   \end{array} \]
   c) \[ \begin{array}{cc}
   x & y \\
   4 & 11 \\
   2 & 14 \\
   0 & 17 \\
   -2 & 20 \\
   -4 & 23 \\
   \end{array} \]
   d) \[ \begin{array}{cc}
   x & y \\
   -2 & -12 \\
   -1 & -5 \\
   0 & 0 \\
   1 & 3 \\
   2 & 4 \\
   \end{array} \]

2. Each table of values represents a linear relation.
   Complete each table. Explain your reasoning.

   a) \[ \begin{array}{cc}
   x & y \\
   1 & \\
   2 & \\
   3 & 14 \\
   4 & 18 \\
   5 & \\
   \end{array} \]
   b) \[ \begin{array}{cc}
   x & y \\
   1 & 1 \\
   3 & 3 \\
   5 & -1 \\
   7 & \\
   \end{array} \]
   c) \[ \begin{array}{cc}
   x & y \\
   4 & \\
   2 & 14 \\
   0 & 19 \\
   -2 & \\
   -4 & \\
   \end{array} \]

3. Create a table of values for each linear relation and then graph the relation.
   Use values of \( x \) from -2 to 2.
   a) \( y = x + 4 \)  
   b) \( y = 2x + 1 \)  
   c) \( y = 5 - 2x \)

4. A computer repair company charges $80 for a service call, plus $50 an hour for labour.
   a) Create a table to show the relation between the time in hours for the service call
      and the total cost.
   b) Is this relation linear? Justify your answer.
   c) Let \( n \) represent the time in hours for the service call and \( C \) represent the total cost in
      dollars. Write an equation that relates \( C \) and \( n \).
   d) How much will a 7-h service call cost?