4.1 Writing Equations To Describe Patterns

**Example 1** Writing an Equation to Represent a Written Pattern

Here is a pattern made with sticks.

- Figure 1
- Figure 2
- Figure 3
- Figure 4
- Figure 5

**a)** Write an equation that relates the number of sticks to the figure number.

**b)** What is the number of sticks in the 10th figure?

**Solution**

**a)** Record the number of sticks in each figure in a table.

<table>
<thead>
<tr>
<th>Figure Number, f</th>
<th>Number of Sticks, n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

As the figure number increases by 1, the number of sticks increases by 2. Repeated addition of 2 is the same as multiplication by 2.

So, the equation $n = 2f$ may represent the relationship.

Check whether the equation is correct.

When $f = 1$, $n = 2(1) = 2$

This is 1 less than 3.

So, add 1.

$2(1) + 1 = 3$

So, an equation is: $n = 2f + 1$

**b)** To find the number of sticks in the 10th figure, substitute $f = 10$ in the equation:

$n = 2f + 1$

$= 2(10) + 1$

$= 20 + 1$

$= 21$

There are 21 sticks in the 10th figure.

$2f + 1$ represents the number of sticks for any figure number $f$.
Check

1. For the table below:

<table>
<thead>
<tr>
<th>Number of Swings, s</th>
<th>Number of Hits, h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

a) Describe the number of hits in terms of the number of swings.
   The number of hits is _____ times the number of swings, plus _____.

b) Write an equation to describe the relationship.
   \[ h = ____ s + ____ \]

c) Use your equation to find \( h \) when \( s = 10 \).
   \[ h = \] ____________

Example 2 Writing an Equation to Represent a Situation

Teagan goes to a carnival.
The cost for a ride is shown on a poster at the entrance.

\[ \text{Entrance Fee } \$5.00 \]
\[ + \]
\[ \$2.00 \text{ per ride} \]

a) Write an equation that relates the total cost, \( C \) dollars, to the number of rides, \( r \).

b) Teagan goes on 4 rides. What is his total cost?
**Solution**

a) The cost is $5.00, plus $2.00 per ride.  
That is, the cost is: \( 5.00 + 2.00 \times (\text{number of rides}) \)  
An equation is: \( C = 5.00 + 2.00r \)

b) Use the equation: \( C = 5.00 + 2.00r \)  
Substitute: \( r = 4 \)  
\( C = 5.00 + 2.00(4) \)  
\( = 5.00 + 8.00 \)  
\( = 13.00 \)  
Teagan's total cost is $13.00.

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**Check**

1. Marcel takes a summer job at a book packaging plant.  
He gets paid $50 a day, plus $2 for every box packed.

![Image of Marcel packing boxes]

a) Write an equation that relates the number of boxes packed to Marcel's pay for a day.  
Let \( P \) represent his pay for one day, and let \( b \) represent the number of boxes packed.  
\( P = \) ________________

b) Marcel packed 20 boxes one day. How much did he get paid?  
\( P = \) ________________  
\( = \) ________________  
\( = \) ________________  
Marcel got paid ______.
1. In each equation, find the value of $T$ when $n = 6$.

   a) $T = 8 + n$
   
   $T = 8 + \underline{\text{_____}}$
   
   $\underline{\text{______}}$
   
   $\underline{\text{______}}$

   b) $T = 3n - 2$
   
   $T = 3\underline{\text{_____}} - 2$
   
   $\underline{\text{______}}$
   
   $\underline{\text{______}}$

   c) $T = 12n + 9$
   
   $T = \underline{\text{______}}$
   
   $\underline{\text{______}}$
   
   $\underline{\text{______}}$

   d) $T = 7n + 3$
   
   $T = \underline{\text{______}}$
   
   $\underline{\text{______}}$
   
   $\underline{\text{______}}$

2. a) This pattern of dots continues. Draw the next 2 figures in the pattern.

   O O O O O O O
   O O O O O O O
   O O O O O O O

   Figure 1   Figure 2   Figure 3   Figure 4

   b) The pattern is represented in a table of values. Which expression below represents the number of dots in terms of the figure number?

   i) $2f$
   ii) $3f$
   iii) $-3f$
   iv) $3f + 1$

<table>
<thead>
<tr>
<th>Figure Number, $f$</th>
<th>Number of Dots, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

3. a) Look at the pattern of tiles below. Draw the next 2 figures in the pattern.

   Figure 1   Figure 2   Figure 3   Figure 4   Figure 5
b) Complete the table below.

<table>
<thead>
<tr>
<th>Number of Shaded Tiles, $s$</th>
<th>Number of White Tiles, $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>__</td>
</tr>
<tr>
<td>__</td>
<td>__</td>
</tr>
<tr>
<td>__</td>
<td>__</td>
</tr>
<tr>
<td>__</td>
<td>__</td>
</tr>
</tbody>
</table>


c) Write an equation for the number of white tiles, $w$, in terms of the number of shaded tiles, $s$.

\[ w = ____s + ____ \]

d) Use your equation to find $w$ when $s = 25$.

\[ w = \ldots \]
\[ = \ldots \]
\[ = \ldots \]

When the number of shaded tiles is 25, there are ____ white tiles.

4. Anabelle is part of the yearbook committee. This year, the set-up cost to print yearbooks is $400, plus $3 for each yearbook printed.

a) Write an equation for the total cost in terms of the number of yearbooks printed.

Let $C$ represent the total cost, and let $n$ represent the number of yearbooks.

\[ C = ____ + ____ \]

b) Anabelle takes 200 orders for yearbooks this year. What is the total cost to the yearbook committee?

\[ C = \ldots \]
\[ = \ldots \]
\[ = \ldots \]

The total cost is $____.
Lesson 4.1: Writing Equations to Describe Patterns

1. In each equation, determine the value of $A$ when $n$ is 3.
   a) $A = 2n + 1$
   b) $A = 3n - 2$
   c) $A = 4n + 3$
   d) $A = 30 - 2n$

2. The pattern in this table continues. Which equation below relates the figure number $n$, to the perimeter of the figure $P$?

<table>
<thead>
<tr>
<th>Figure Number, $n$</th>
<th>Perimeter, $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

   a) $P = 3n + 7$
   b) $P = 7n + 3$
   c) $P = 3n + 4$
   d) $n = 3P + 7$

3. The pattern in each table below continues. For each table:
   i) Describe the pattern that relates $v$ to $t$.
   ii) Write an equation that relates $v$ to $t$.
   iii) Verify your equation by substituting values from the table.

   a) 
<table>
<thead>
<tr>
<th>Term Number, $t$</th>
<th>Term Value, $v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>23</td>
</tr>
</tbody>
</table>

   b) 
<table>
<thead>
<tr>
<th>Term Number, $t$</th>
<th>Term Value, $v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
</tbody>
</table>

4. Rachel takes care of homes during the summer while their owners are away on vacation. She charges $8, plus $2.50 a day.
   a) Create a table that shows the charges when the owners are away for up to 5 days.
   b) Write an equation that relates the charge, $C$ dollars, to the number of days, $n$, that the owners are away.
   c) What will the charge be when the owners are away for 14 days?
   d) How many days were the owners away when the charge was $33?